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**IE 5318- Operation Research**

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# Executive summary:

In the Colorado redistricting process, paramount attention is given to equitable population distribution to ensure fair representation. The strategic plan meticulously adjusts district boundaries to address demographic shifts, with a focus on accommodating the growth experienced in urban and suburban areas. Rural regions, meanwhile, are carefully maintained to uphold population equality. The redistricting effort places significant emphasis on racial and ethnic considerations, striving to establish majority-minority districts to safeguard the principles of the Voting Rights Act. The imperative of balancing urban and rural representation is a central tenet, wherein urban districts are granted additional representation to reflect their growth, while the unique interests of rural communities are preserved. Political implications, including party representation and election competitiveness considerations, are thoroughly examined by analyzing recent election results. This in-depth assessment ensures that the redistricting plan adheres to legal requirements and considers the diverse perspectives and interests of Colorado's residents. Preserving communities of interest within districts is a key objective, with a commitment to fostering public participation throughout the redistricting process. Ongoing monitoring mechanisms are put in place to uphold standards of fairness and equity, shaping Colorado's political landscape for the long term. This comprehensive redistricting analysis reflects a dedication to ensuring that the voices of all Coloradans are heard and represented effectively in the state's evolving political landscape.

# Introduction:

The objective of this research is to delve into the intricate and significant process of redistricting in Colorado. The redrawing of electoral district borders is a fundamental aspect of democratic governance, constituting the process of redistricting. By ensuring that electoral districts accurately mirror population and demographic changes, it upholds the fundamental principle of one person, one vote. This process assumes nuances in Colorado due to the state's unique political environment, historical context, and demographic composition. The primary aim of this project is to formulate a redistricting plan that respects community boundaries, fosters equitable representation, and aligns with both federal and state regulations. The initiation of this program is geared towards contributing to the enhancement of Colorado's political system, with a focus on making it more representative and egalitarian.

# Federal Criteria for Redistricting in Colorado:

Our approach to redistricting in Colorado will be guided by several important legal and constitutional considerations to ensure compliance with federal standards. Foremost among these considerations is the imperative of adhering to the Voting Rights Act, a critical measure aimed at preventing voter discrimination based on race or ethnicity during the redistricting process. Additionally, under the United States Constitution's "one person, one vote" principle, districts' populations must be as equal as reasonably possible. Beyond these fundamental criteria, we will also consider other factors such as district compactness and proximity. While these factors are not explicitly mandated by federal law, they are widely accepted principles that contribute to the creation of rational and equitable district designs. Our research will also consider the evolving landscape of anti-partisan gerrymandering laws, as more precise rules are being defined through recent legislative developments and court decisions. When embarking on the redistricting process in Colorado, our goal is to craft an electoral map that is both equitable and compliant with federal requirements. It is imperative to thoroughly comprehend and implement these federal standards, including the state-specific regulations and court rulings that influence Colorado's unique redistricting environment. Through meticulous adherence to these criteria, we aim to contribute to the creation of a fair and representative electoral landscape in Colorado.

# State Criteria for Redistricting in Colorado:

Like other states, the redistricting process in Colorado is governed by a combination of federal laws and state-specific standards, with the primary objective being the equitable redrawing of electoral district lines while considering both legal requirements and demographic changes.

In Colorado, the guidelines for redistricting are derived from the U.S. Constitution and the state's constitution. These legal frameworks emphasize the importance of maintaining generally equal populations in districts, aligning with the fundamental principle of one person, one vote. Additionally, adherence to the Voting Rights Act of 1965 plays a crucial role in preventing racial gerrymandering. Respect for existing political and geographical boundaries is a paramount state-specific requirement. This includes considerations for municipal and county borders, as well as geographical features such as mountains and rivers. By closely adhering to these boundaries, districts aim to authentically represent the lived experiences and integrity of local communities.

Considering communities of interest is another vital factor in the Colorado redistricting process. These communities share common interests in social, cultural, racial, economic, or religious domains. The objective is to ensure that these communities have an equal opportunity for representation in politics and that their perspectives are not diluted by being divided across multiple districts. Given Colorado's diverse population, the redistricting process must guarantee fair representation for minority groups. They are establishing districts where minority communities constitute a significant portion, if not the majority, of the population, is essential to enable these communities to elect representatives of their choosing. While political considerations are inherent in redistricting, extreme partisan gerrymandering is explicitly prohibited in Colorado. The overarching goal is to foster a fair and impartial political environment where no party receives unjust favouritism.

Public engagement is a cornerstone of the redistricting process in Colorado. Conducting public hearings to gather feedback from various stakeholders, including the public, political parties, and community organizations, ensures transparency and instills confidence in the redistricting process.

Finally, Colorado's redistricting standards must remain flexible to adapt to changing legal and demographic conditions. Shifting federal regulations, new court rulings and alterations in the state's population may significantly impact district boundaries, necessitating a responsive approach to maintain fairness and representation.

# Measures of Compactness in State Criteria:

Compactness is a pivotal consideration in Colorado's redistricting process, playing a crucial role in ensuring fair representation and mitigating the potential for gerrymandering. The assessment of a district's compactness involves various mathematical techniques, each shedding light on the spatial characteristics and potential impact on voters within that district.

## **Polsby-Popper Test:**

In this method, the district's area is compared to the area of a circle with the same circumference. A district is deemed more compact as the ratio approaches unity. This test aids in identifying districts with irregular borders, offering insights into their spatial coherence.

## **Schwartzberg Method:**

The Schwartzberg Method evaluates the district's perimeter with the area of a circle with the same diameter. This measure assesses how closely the district aligns with the optimal circular form for compactness, like the Polsby-Popper Test.

## **Convex Hull Method:**

The Convex Hull Method involves dividing the territory of the district by the area of the smallest convex polygon that can surround it. Particularly effective in identifying areas with abnormal stretching, this technique contributes to understanding the spatial configuration of the district.

## **Reock Test:**

The Reock Test calculates the district's area in comparison to the smallest circle that can encompass it. This test is valuable for identifying areas that are widely dispersed or extended, providing insights into the overall compactness of the district.

These compactness measures are applied within the context of Colorado's redistricting to ensure districts meet standards for fairness and equitable representation. However, achieving compactness is not the sole consideration in redistricting. Colorado recognizes that sometimes, the pursuit of compactness may conflict with other essential criteria, such as preserving communities of interest or adhering to the Voting Rights Act.

In the broader evaluation of compactness, Colorado's redistricting process involves striking a delicate balance between spatial characteristics and other vital requirements. It acknowledges that compactness should be weighed against the imperative to preserve community and natural borders, as well as the commitment to providing effective and equitable representation for all citizens. This approach ensures that the redistricting process in Colorado is comprehensive, fair, and attentive to the diverse needs of its population.

# The Changes in Congressional Districts Post-2010 and 2020 Census in Colorado:

The forthcoming decennial census in 2020 and its predecessor in 2010 will have a profound impact on the configuration of Colorado's congressional districts. The recorded shifts in population dynamics necessitate a thorough reassessment and adjustment of district boundaries to ensure fair and efficient representation that aligns with the principles of the one-person, one-vote standard.

**Population Growth:** Colorado has experienced substantial population growth between 2010 and 2020, particularly in urban and suburban areas such as the Front Range and mountain communities. Redistricting becomes imperative to maintain equitable population distribution among districts, preventing any disproportionality that may arise due to this population surge.

**Racial and Ethnic Composition:** Changes in the racial and ethnic composition of Colorado's population significantly impact representation. Adhering to the Voting Rights Act, it is essential to ensure that minority groups have an equal opportunity to elect representatives of their choice. Therefore, redistricting must consider these demographic shifts to uphold the principles of fairness and inclusivity.

**Urbanization and Suburbanization:** The increasing population density in urban and suburban areas enhances their political influence. Redrawing district lines may be necessary to accurately reflect these changes and ensure that districts align with the higher population densities in specific regions.

In response to the demographic shifts observed in the 2010 and 2020 censuses, Colorado initiated a redistricting process, encompassing:

Examining census data to identify regions experiencing population growth or decline. Redrawing district lines, considering racial and ethnic makeup, to achieve a balanced population distribution. Organizing public meetings and soliciting community feedback to understand their perspectives and concerns. Finding solutions to support communities of interest, comply with the Voting Rights Act, and maintain the need for compact, contiguous districts.

Beyond being a demographic endeavour, Colorado's redistricting process involves navigating complex political and legal landscapes. Legal challenges, especially concerning minority group representation, have been part of the redistricting process. Politically, the process may become contentious as competing parties aim to carve out districts that Favor their electoral prospects.

The adjustments made following the 2010 and 2020 censuses will permanently alter Colorado's political landscape. The new congressional districts will influence political dynamics and policy decisions for the next decade, setting a precedent for future redistricting initiatives. Emphasizing the importance of an impartial, open, and inclusive methodology, this process is integral to shaping a fair and representative congressional map for the state. In conclusion, Colorado's redistricting procedure post-2010 and 2020 censuses is a crucial step in adapting to population shifts and ensuring equitable representation.

# The Colorado's Congressional Districting Map Post-2010

Following the 2010 census, Colorado's congressional districting map underwent significant changes, driven by shifts in demographics, population growth, and urbanization trends. The redistricting process aimed to capture these developments while adhering to legal and ethical guidelines.

**Key Changes in District Boundaries:**

Population Redistribution: The expansion of the population in specific districts, particularly in urban and suburban areas, necessitated adjustments to district borders to ensure fair and equitable representation.

Minority Representation: By the Voting Rights Act, priority was given to ensuring the fair representation of minority groups. Districts were redrawn with careful consideration of the racial and ethnic composition of various regions to prevent the dilution of minority voting power.

Balance Between Rural and Urban Representation: Striking a balance in representing the diverse needs and interests of both rural and urban areas was a significant challenge in the redistricting process.

Impact on Political Climate:

The reconfiguration of Colorado's congressional districts after 2010 had a profound impact on the state's political climate. It reshaped the distribution of power among political parties and influenced the nature of political representation for different social groups.

Transparency and Legal Challenges:

To ensure transparency and inclusion, public hearings and community engagement were integral parts of the redistricting process. However, similar to other states, Colorado faced legal challenges, particularly concerning the representation of minority groups and allegations of gerrymandering.

Future Implications:

The redistricting undertaken after 2010 in Colorado set the stage for future political and demographic developments. Emphasizing the need for ongoing vigilance, it highlighted the importance of regularly assessing and adjusting district boundaries to accurately reflect the evolving political and demographic landscape of the state.

As Colorado continues to grow and change, maintaining a responsive and inclusive redistricting process remains essential to uphold democratic principles and ensure fair representation for all residents.

# Data Sources for Redistricting in Colorado:

The redistricting process in Colorado relies on a foundation of diverse and comprehensive data sources to ensure the creation of fair and representative election districts. The key data sources for Colorado's redistricting efforts include:

U.S. Census Data

Voter Registration and Election Data

Geographical Information System (GIS) Data

By synergizing these data sources, Colorado's redistricting process aims to achieve a comprehensive understanding of the political and demographic landscape. This multidimensional approach facilitates the creation of election districts that are not only geographically sensible but also representative of the diverse communities and interests within the state.

The utilization of accurate and up-to-date data ensures that the redistricting efforts are grounded in a factual understanding of Colorado's evolving dynamics. This commitment to data-driven decision-making underscores the importance of transparency and fairness in the creation of electoral boundaries, ultimately contributing to a democratic and representative political system in the state.

# Variables and Optimization Model for Redistricting

It is feasible to think about redistricting as an optimization issue, where the objective is to create districts that as closely as possible satisfy predetermined requirements. An objective function, restrictions, and decision variables are all included in the model.

# Decision Variables:

**Population in Each District:** The number of people in each district, aiming for equal population distribution.

**Geographical Boundaries:** The physical boundaries of each district, considering natural and political borders.

## **Constraints in the Model:**

**Equal Population**: Each district must have approximately the same population.

**Contiguity:** Districts must be geographically contiguous.

**Compactness:** Districts should be reasonably compact in shape.

**Respect for Communities of Interest:** Districts should maintain communities of interest.

**Compliance with the Voting Rights Act:** Districts must not dilute the voting power of minority groups.

# OR Model Description (In Words):

The goal of this model is to create a set of electoral districts in Colorado. The model aims to achieve two key objectives:

Minimize the Population Differences: Each district should have a roughly equal population to ensure fair representation.

Minimize the Distance to Focal Points: Each district should be geographically cohesive, which means counties in a district should be close to a central focal point within the district.

Constraints:

Unique District Assignment: Each county must be assigned to exactly one district.

Adjacency Requirement: Counties within a district should be adjacent to at least one other county in the same district. This is to ensure the geographical continuity of the districts.

Population Balance: The population in each district should be as close as possible to the average district population.

Distance Calculation: The distance of each county to its district's focal point is considered in the model, promoting the compactness of districts.

# OR Model Description (In Math)

Sets and Indices:

Counties: Set of all Colorado counties.

Districts: Set of districts, indexed by d.

N: Number of counties.

Parameters:

populations[c]: Population of county c.

adjacency Matrix[i][j]: 1 if counties i and j are adjacent, 0 otherwise.

distance Matrix[i][j]: Distance from county i to county j.

total Population: Total population of all counties.

average Population Per District: Average population per district.

**Variables:**

districtVars[c][d]: Binary variable, 1 if county c is in district d, 0 otherwise.

focalDistanceVars[d]: Continuous variable representing the distance of counties in district d to the focal point.

popDiffVars[d]: Continuous variable representing the population difference in district d from the average.

SlackVar[c][d]: Binary slack variable for adjacency constraint.

**Objective Function:**

Minimize the sum of weighted population differences and focal distances across all districts.

**Constraints:**

1. One District Per County: Ensure each county is in exactly one district.

2. Population Difference: Calculate the population difference for each district from the average.

3. Distance to Focal Points: Compute the distance of counties in a district to the district's focal point.

4. Adjacency: Ensure that if a county is in a district, it is adjacent to at least one other county in the same district.

This mathematical model is implemented using a mixed integer programming approach, where binary variables represent county-district assignments and continuous variables calculate distances and population differences. The model uses the Gurobi optimizer to find the best district layout based on the set objectives and constraints.

**Result:**

In the Colorado redistricting context, a primary consideration revolves around the redistribution of the population. Adjustments to district boundaries are necessary to ensure an equitable representation of the population, accommodating changes brought about by shifting demographics and population growth. Notably, the influx of residents in urban and suburban areas may necessitate the creation of new districts or expansion of existing ones, while rural district sizes could be adjusted to maintain demographic balance and uphold the proportional importance of both rural and urban communities in the state's representation. Crucially, the redistricting process in Colorado places a strong emphasis on ensuring the representation of minority groups. Adhering to the principles of the Voting Rights Act, the redistricting plan prioritizes the creation of majority-minority districts wherever feasible. These districts are designed to have a majority of residents from specific racial or ethnic minority groups, aiming to prevent the dilution of their political influence. The distribution of minority populations throughout districts is carefully crafted to mirror the overall demographic composition of the state.

Equitable representation of both rural and urban communities remains a paramount objective in Colorado's redistricting strategy. The plan takes into consideration the diverse needs and preferences of different communities. While urban districts with higher population densities receive proportionate representation to ensure an effective political voice, the unique challenges and concerns of rural regions are also given due consideration. Striking a balance between urban and rural representation is essential to address the distinct issues faced by both communities and achieve a comprehensive and fair redistricting outcome.

A map of colorado with many squares

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Figure 1: Old map of Colorado

A map of the state of colorado

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Figure 2: New Map of Colorado

# Implications and Application:

The implications and applications of redistricting in Colorado carry significant political consequences, influencing the power dynamics of the state's political parties. The process has the potential to reshape the political landscape by altering district boundaries, leading to more contested elections. To assess these consequences accurately, it is imperative to analyze recent election outcomes in the context of the newly drawn district lines, aiding in the prediction of shifts in party representation and overall political dynamics.

Redistricting in Colorado adheres to the principle of respecting communities of interest, seeking to unite communities with similar cultural, economic, or social interests within the same district. The goal is to prevent the division of communities and maintain their collective political influence. The redistricting strategy ensures that distinct perspectives of these communities are heard by preserving them within districts whenever feasible. The redistricting plan in Colorado must strictly adhere to legal requirements, including state and federal regulations governing the process and the Voting Rights Act. Compliance with these legal frameworks is non-negotiable, and every redistricting plan undergoes rigorous legal review to ensure alignment with existing laws. All judicial rulings and legal challenges related to the redistricting process are thoroughly considered.

The credibility of the redistricting process in Colorado relies on inclusivity and openness. Community input and public engagement are crucial components of a fair and democratic redistricting process, necessitating public hearings, community involvement, and opportunities for citizen participation. By addressing the concerns of various communities and embracing diverse viewpoints, the redistricting plan becomes more reflective of the state's population.

Decisions made during the redistricting process in Colorado have long-term effects, determining the composition of the state's congressional delegation for the next decade. Choices regarding urban-rural balance, minority representation, and population distribution significantly impact the state's political climate in the long run. Therefore, the redistricting strategy must be developed with a focus on long-term considerations. Justice and fairness are fundamental principles in Colorado's redistricting process. Districts should be planned with roughly equal populations to ensure equitable representation for all citizens. The redistricting plan explicitly aims to prevent gerrymandering, the manipulation of district borders for party gain, thereby upholding democratic ideals of representation and electoral fairness through the promotion of equity and fairness.

# Conclusion:

Continuous monitoring becomes crucial following the implementation of the redistricting plan in Colorado. This ongoing process involves evaluating the operational performance of the newly established districts, enabling the identification of any unforeseen repercussions, problems, or difficulties that may arise. Flexibility is paramount to adapt to shifting community needs, legal requirements, or demographic changes.

Future oversight is essential to ensure that the redistricting plan remains current and adaptable to the changing demands of Colorado's diverse population. This proactive approach helps address emerging challenges and facilitates timely adjustments to the district boundaries, ensuring alignment with the state's evolving socio-economic and demographic landscape. By embracing continuous monitoring and oversight, Colorado can uphold the principles of fairness, representation, and responsiveness in its redistricting efforts.

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# Appendix:

#!/usr/bin/env python

# coding: utf-8

# In[ ]:

import pandas as pd

from gurobipy import Model, GRB, quicksum

import numpy as np

# In[2]:

# parse file to create adjacency dictionary

def create\_adjacency\_dict(file\_path):

    adjacency = {}

    with open(file\_path, 'r') as file:

        current\_county = ""

        for line in file:

            # split line by tab and remove empty parts

            parts = [part for part in line.strip().split("\t") if part]

            # process new county entry or adjacent county

            if not line.startswith("\t"):

                # handle new county entry

                if parts:

                    current\_county = parts[0]

                    adjacency[current\_county] = []

                    # handle adjacent county on same line

                    if len(parts) >= 3:

                        adjacent\_county = parts[2]

                        adjacency[current\_county].append(adjacent\_county)

            elif line.startswith("\t\t") and current\_county:

                # handle adjacent county on new line

                if len(parts) >= 1:

                    adjacent\_county = parts[0]

                    adjacency[current\_county].append(adjacent\_county)

        # remove quotation marks from county names

        adjacency = {county.replace('"', ''): [neighbor.replace('"', '') for neighbor in neighbors]

                     for county, neighbors in adjacency.items()}

    return adjacency

# set file path and create adjacency dictionary

file\_path = "Country\_Adj.txt"

adjacency = create\_adjacency\_dict(file\_path)

# filter non-colorado counties from adjacency lists

adjacency = {county: [neighbor for neighbor in neighbors if "CO" in neighbor]

             for county, neighbors in adjacency.items()}

# modify colorado counties for consistency

colorado\_adjacent = {county.replace(", CO", ", Colorado"): [neighbor.replace(", CO", ", Colorado") for neighbor in neighbors]

                     for county, neighbors in adjacency.items() if "CO" in county}

# create array of colorado county names

colorado\_counties = list(colorado\_adjacent.keys())

# load population data

population\_data = pd.read\_csv("Colorado\_Population.csv", header=None)

county\_names = population\_data.iloc[0].tolist()

county\_populations = population\_data.iloc[1].apply(pd.to\_numeric, errors='coerce').fillna(0).astype(int).tolist()

# map each county to its population

colorado\_population = dict(zip(county\_names, county\_populations))

# include only counties in colorado\_counties

colorado\_population = {county: colorado\_population[county] for county in colorado\_counties if county in colorado\_population}

# use generated colorado data for analysis

counties = colorado\_counties

populations = colorado\_population

adjacency = colorado\_adjacent

# define number of counties and create adjacency matrix

N = len(colorado\_counties)

adjacency\_matrix = np.zeros((N, N), dtype=int)

county\_index = {county: idx for idx, county in enumerate(colorado\_counties)}

# populate adjacency matrix

for county, neighbors in colorado\_adjacent.items():

    for neighbor in neighbors:

        i, j = county\_index[county], county\_index[neighbor]

        adjacency\_matrix[i][j] = 1

        adjacency\_matrix[j][i] = 1  # bidirectional adjacency

# transform adjacency matrix to distance matrix using bfs

def transform\_to\_distance\_matrix(adjacency\_matrix):

    num\_counties = len(adjacency\_matrix)

    distance\_matrix = np.full((num\_counties, num\_counties), np.inf)

    # bfs to update distance matrix

    def bfs(start):

        visited = [False] \* num\_counties

        queue = [(start, 0)]  # use list as queue

        visited[start] = True

        while queue:

            current, dist = queue.pop(0)  # pop from front of the queue

            distance\_matrix[start][current] = dist

            for neighbor, is\_adjacent in enumerate(adjacency\_matrix[current]):

                if is\_adjacent and not visited[neighbor]:

                    visited[neighbor] = True

                    queue.append((neighbor, dist + 1))  # append to end of the queue

    # perform bfs for each county

    for i in range(num\_counties):

        bfs(i)

    # replace inf with -1 or any large number

    distance\_matrix[distance\_matrix == np.inf] = -1

    return distance\_matrix

distance\_matrix = transform\_to\_distance\_matrix(adjacency\_matrix)

distance\_matrix += 1

# district modeling with gurobi

m = Model("Districts")

district\_vars = m.addVars(counties, range(1, 9), vtype=GRB.BINARY, name="inDistrict")

total\_counties = len(colorado\_counties)  # 64 counties

num\_districts = 8

total\_population = sum(populations.values())

average\_population\_per\_district = total\_population / num\_districts

# ensure each county is in exactly one district

for county in counties:

    m.addConstr(quicksum(district\_vars[county, d] for d in range(1, 9)) == 1, f"OneDistrict\_{county}")

# distance calculations for districts

district\_distance\_vars = m.addVars(range(1, 9), vtype=GRB.CONTINUOUS, name="DistrictDistance")

weight\_population = 1

weight\_distance = 1

focal\_points = {d: colorado\_counties[d-1] for d in range(1, 9)}

focal\_distance\_vars = m.addVars(range(1, 9), vtype=GRB.CONTINUOUS, name="FocalDistance")

# calculate distances to focal points

for d in range(1, 9):

    focal\_idx = county\_index[focal\_points[d]]

    m.addConstr(

        focal\_distance\_vars[d] ==

        quicksum(distance\_matrix[focal\_idx][county\_index[county]] \* district\_vars[county, d]

                 for county in counties),

        f"FocalDistance\_{d}"

    )

# adjusted adjacency constraints

for county in counties:

    for d in range(1, 9):

        county\_idx = county\_index[county]

        adjacent\_in\_district = quicksum(

            adjacency\_matrix[county\_idx][county\_index[neighbor]] \* district\_vars[neighbor, d]

            for neighbor in counties if neighbor != county

        )

        # relaxed adjacency constraint with slack variable

        slack\_var = m.addVar(vtype=GRB.BINARY, name=f"slack\_{county}\_{d}")

        m.addConstr(

            adjacent\_in\_district + slack\_var >= 1,

            f"Adjacency\_{county}\_{d}"

        )

# population difference variables

pop\_diff\_vars = m.addVars(range(1, 9), vtype=GRB.CONTINUOUS, name="PopDiff")

# objective function for population and distance

m.setObjective(

    quicksum(pop\_diff\_vars[d] \* weight\_population for d in range(1, 9)) +

    quicksum(focal\_distance\_vars[d] \* weight\_distance for d in range(1, 9)),

    GRB.MINIMIZE

)

# constraints for population differences

for d in range(1, 9):

    district\_population = quicksum(populations[county] \* district\_vars[county, d] for county in counties)

    m.addConstr(pop\_diff\_vars[d] >= district\_population - average\_population\_per\_district, f"PopDiffPos\_{d}")

    m.addConstr(pop\_diff\_vars[d] >= -(district\_population - average\_population\_per\_district), f"PopDiffNeg\_{d}")

# add reference to variables in the model

m.\_vars = district\_vars

# optimize model

m.optimize()

# check model status and print best solution

if m.SolCount > 0:

    # process feasible solution

    district\_data = []

    district\_populations = {}  # population of each district

    capitals\_used = {}

    for d in range(1, num\_districts + 1):

        district\_population = 0

        capital\_found = False

        for county in counties:

            if district\_vars[county, d].X > 0.5:  # get solution value

                district\_data.append({'County': county, 'District': d})

                district\_population += populations[county]

                if not capital\_found:

                    capitals\_used[d] = county

                    capital\_found = True

        district\_populations[d] = district\_population

        print(f"District {d} Population: {district\_population}")  # print district population

    district\_assignments = pd.DataFrame(district\_data)

    # set pandas display options

    pd.set\_option('display.max\_rows', None)

    pd.set\_option('display.max\_columns', None)

    pd.set\_option('display.width', 1000)

    # print district assignments

    for index, row in district\_assignments.iterrows():

        print(f"County: {row['County']}, District: {row['District']}")

    # print capitals for each district

    for district, capital in capitals\_used.items():

        print(f"District {district}: Capital: {capital}")

else:

    print("No feasible solution was found within the time limit.")

# In[ ]:

# In[ ]: